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5. Risk Measures

VaR is one of the most commonly used risk measures which reflects the maximum loss within a given confidence level (Jorion, 2001). Despite its simplicity, VaR is not coherent because of not fulfilling subadditivity property except under the normal distribution assumption.

VaR can be expressed as

$$\mathrm{VaR}_q(X) = F_X^{-1}(q) = \sup\{x\,|\mathbb{P}(X > x) > q\}$$

where F^{-1} is the *quantile function* of random variable X defined as the inverse of the distribution function F. Although it would be better to present both VaR and expected shortfall to compare the models using different risk measures, we only calculate VaR to provide a benchmark comparison to keep the simplicity. However, we will consider the other risk measures for future studies.

The methods used for computing VaR can be grouped as the parametric and non-parametric approaches. In this paper, we estimate VaR values for EVT and GARCH models both of which are parametric approaches. In addition to the above definition that is used to calculate VaR for the selected EVT models, we use another calculation of VaR suggested by Genay *et al.* (2003). This definition, which is proposed as a variance covariance approach, is useful for our study due to its applicability to GARCH models.

Let $r_t, t = 1, 2, \ldots, n-1$ is the log-return of the prices which follows a martingale process as $r_t = \mu_t + \epsilon_t$ where ϵ_t has a distribution function F with zero mean and variance σ_t^2. The VaR in this case can be calculated as

$$\mathrm{VaR}_t(\alpha) = \hat{\mu}_t + F^{-1}(\alpha)\hat{\sigma}_t$$

where $F^{-1}(\alpha)$ is the q-th quantile $(q = 1 - \alpha)$ value of the unknown distribution function F. An estimate of μ_t and σ_t^2 can be obtained from the sample mean and the sample variance. Instead of the sample variance, the standard deviation in this equation can also be estimated by a statistical model (Gençay *et al.*, 2003).

According to the optimal EVT and GARCH models, we estimate the VaR of developed and emerging countries for positive and negative log-returns, respectively. Figures 3 and 4 display the calculated VaR values of the chosen methods for each country for 95% and 99.5% confidence levels.

Figure 3. Risk measure results for positive log-returns for developed and emerging countries

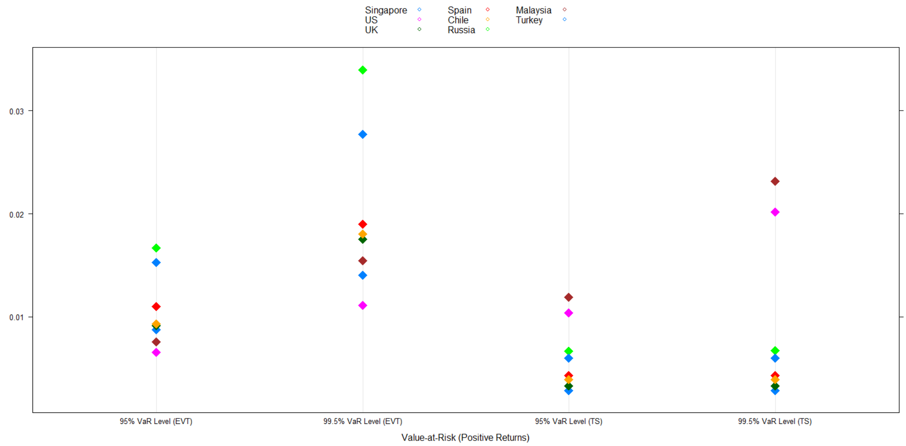

Figure 4. Risk measure results for negative log-returns for developed and emerging countries

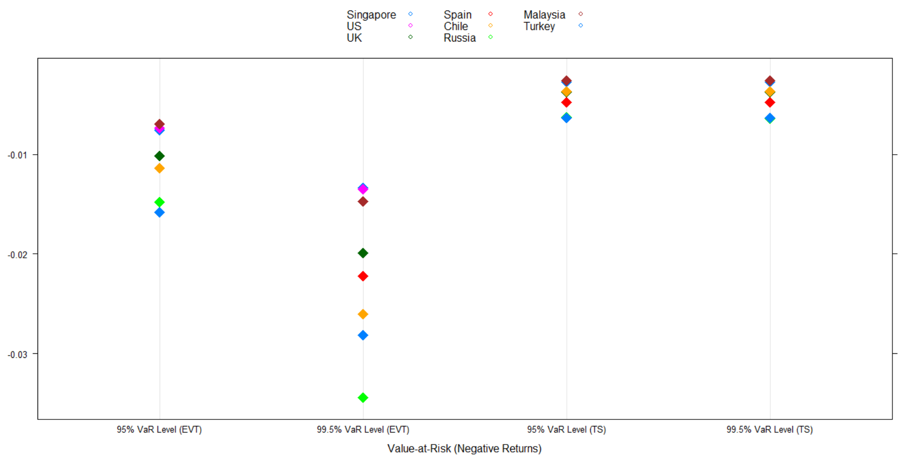

Figure 3 shows that the risk measures obtained from the EVT models are generally higher than the risk measures obtained from the time series models for positive returns which indicates that the EVT models provide more conservative results. The ranking of the countries according to the VaR are consistent within the same model (EVT or time series) for different confidence levels (95% and 99.5%) except for a few discrepancies.

Figure 3 displays the VaR analysis for positive returns of the EVT models, which shows that Russia and Turkey have the highest risk measures and the US has the smallest. However, the VaRs obtained from the time series models indicate that Malaysia and the US have the highest risk measures while Singapore has the smallest. Different models might lead different country rankings in terms of risk measures. Moreover, the graphs do not suggest a significant boundary between the developed and emerging markets which makes it difficult to analyse the relative positions of the countries.

By analysing Figure 4 we see that the absolute value of the VaR results are higher compared to the positive returns for both EVT and time series models. The rankings of the countries based on the calculated risk measures for two sets of models are different. Time series models are more consistent based on the country ranking for two different confidence levels while the EVT models indicate some discrepancies. Based on the negative returns data, EVT models show that Russia and Turkey have the highest risk while Malaysia and Singapore have the lowest. Time series models for two confidence levels present almost identical numbers for the risk measures which change on the fifth or sixth decimal places and also indicates Russia and Turkey are the most risky countries while the US and Malaysia are the least. There is no indication of a separation of the developed and emerging countries in terms of the risk measures obtained for both sets of models.

The results obtained for different confidence levels might indicate that the structure of the tail distributions of these countries are similar due to the increments on the risk measures between different confidence levels.

When we compare the rankings of these countries based on HDI and the riskiness of their stock markets, we see that there is a correlation between those two. Higher HDI indicates lower riskiness for emerging markets and the countries have been ordered from the highest to lowest HDI as Chile, Russia, Malaysia and Turkey. However, the ranking is not that strict for the developed countries since Spain has the second lowest VaR just after Singapore while it is the fourth in terms of the HDI index. Table 13 shows the rankings of the countries for both the HDI and the VaR obtained from the analysis of the returns.

Table 13: Rankings of the countries in terms of HDI and VaR

Country	HDI Ranking	VaR Ranking
Singapore	1	1
USA	2	4
UK	3	3
Spain	4	2
Chile	5	5
Russia	6	6
Malaysia	7	7
Turkey	8	8

The rankings presented in Table 13 are consistent with the graphs of the prices and the log-returns displayed in Figure 1 and 2. Since the only difference in ranking exists for the US and Spain, the log-return graph of the US confirms the relative riskiness of its stock markets due to the higher volatility.

6. Conclusions

We have compared developed and emerging countries based on the HDI index and the VaR results obtained from EVT and GARCH and ARMA-GARCH models. The results show that the time series models fit the log-return data better for all the countries chosen (Singapore, Spain, UK, US, Chile, Russia, Malaysia and Turkey) based on the BIC values. The risk measures obtained from EVT models are more conservative with respect to the risk measures obtained from the time series models. They are also sensitive to the confidence levels due to the changing rankings of the countries based on the calculated risk measures. However, the time series models provide more consistent results for both negative and positive log-returns. Due to the increments of the risk measures for 95% and 99.5% confidence levels, the tail structures are similar for all countries. The risk measures obtained from different models for different confidence intervals might indicate different rankings which makes us to consider more sophisticated ranking/ordering approaches. Although the HDI rankings and the VaR rankings seem consistent for most of the countries, the main components of the HDI which are life expectancy index, education index and gross national income per capita might have different affects on the rankings with different weights. We intend to consider the stochastic ordering approaches to compare the countries based on different risk measures and

HDI for further studies.

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